SEM-2 (MATHEMATICS GENERAL)

ANUP RAY

Diff. Eqns.

Answer of Home work

1) $\tan^{-1}\frac{y}{x} = x + c$ 2) $\frac{1}{xy} = \frac{1}{x}\log x + \frac{1}{x} + c$

3) $\sqrt{x^2+y^2} + ax = c$ 4) $\frac{x^2}{2} + \frac{y^2}{2} + \tan^{-1}\frac{y}{x} = c$

5) $xe^y + y^2 = c$

Rule 2 If $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$, a function of x-alone, then $e^{\int f(x)\,dx}$ is an integrating factor of $Mdx + Ndy = 0$

Ex: Solve $(x^2+y^2+x)\,dx + xy\,dy = 0$

Ans: Comparing the given equation with $Mdx + Ndy = 0$ we get

$M = x^2+y^2+x$, $N = xy$

$\therefore \frac{\partial M}{\partial y} = 2y$ $\frac{\partial N}{\partial x} = y$

$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$, so the given equation is not exact.

Now $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2y-y}{xy} = \frac{1}{x}$, which is a function of x-alone.

$\therefore$ IF $= e^{\int \frac{1}{x}dx} = e^{\log x} = x$

Multiplying both sides of given equation by I.F. (ie x) we get

$(x^3+xy^2+x^2)\,dx + x^2y\,dy = 0$ which must be exact Equation

$\therefore \int M\,dx = \int (x^3 + xy^2 + x^2)\,dx$

$= \frac{x^4}{4} + \frac{x^2y^2}{2} + \frac{x^3}{3}$

{Taking y as constant}

$\int$ (terms in N not containing x) $dy = 0$

$\therefore$ The general solution of the given equation is

$\frac{x^4}{4} + \frac{x^2y^2}{2} + \frac{x^3}{3} = c$ where c is an arbitrary constant.

<u>Home work</u>

Solve:

1) $(x^3 - 2y^2)\,dx + 2xy\,dy = 0$

2) $(x^2 + y^2)\,dx - 2xy\,dy = 0$

3) $(x^2 + y^2 + 1)\,dx + x(x - 2y)\,dy = 0$

4) $(x^2 + y^2 + 2x)\,dx + 2y\,dy = 0$